SOLUTION OF EXERCISE # 2.2

Exercise # 2.2

Q.1: Expand up to four terms.

(i) $\dfrac{1}{\sqrt{1+x}}$

Sol. $\dfrac{1}{\sqrt{1+x}}=(1+x)^{-\frac{1}{2}}$

> Put $b = x$ & $n = -\frac{1}{2}$ in Binomial series Formula,
> $(1+b)^n = 1 + nb + \dfrac{n(n-1)}{2!}b^2 + \dfrac{n(n-1)(n-2)}{3!}b^3 + \dots$
> we have :

$$(1+x)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)(x)+\frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2!}(x)^2+\frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}(x)^3+\dots$$

$$=1-\frac{x}{2}+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)x^2+\frac{1}{6}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)x^3+\dots$$

$$=1-\frac{x}{2}+\frac{3}{8}x^2-\frac{15}{48}x^3+\dots=\boxed{1-\frac{x}{2}+\frac{3}{8}x^2-\frac{5}{16}x^3+\dots}$$

(ii) $(1-2x)^{-\frac{3}{4}}$

Sol. $(1-2x)^{-\frac{3}{4}}$

> Put $b = -2x$ & $n = -\frac{3}{4}$ in Binomial series Formula,
> $(1+b)^n = 1 + nb + \dfrac{n(n-1)}{2!}b^2 + \dfrac{n(n-1)(n-2)}{3!}b^3 + \dots$
> we have :

$$(1-2x)^{-\frac{3}{4}}=1+\left(-\frac{3}{4}\right)(-2x)+\frac{-\frac{3}{4}\left(-\frac{3}{4}-1\right)}{2!}(-2x)^2+\frac{-\frac{3}{4}\left(-\frac{3}{4}-1\right)\left(-\frac{3}{4}-2\right)}{3!}(-2x)^3+\dots$$

$$=1+\frac{3}{2}x+\frac{1}{2}\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)(4x^2)+\frac{1}{6}\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)\left(-\frac{11}{4}\right)(-8x^3)+\dots$$

$$=1+\frac{3}{2}x+\frac{84}{32}x^2+\frac{12936}{384}x^3+\dots=\boxed{1+\frac{3}{2}x+\frac{21}{8}x^2+\frac{77}{16}x^3+\dots}$$

SOLUTION OF EXERCISE # 2.2

(IIA-2017)

(iii) $(1+x)^{-3}$

Sol. $(1+x)^{-3}$

Put $b = x$ & $n = -3$ in Binomial series Formula,
$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots$$
we have :

$$(1+x)^{-3} = 1 + (-3)(x) + \frac{-3(-3-1)}{2!}(x)^2 + \frac{-3(-3-1)(-3-2)}{3!}(x)^3 + \ldots$$

$$= 1 - 3x + \frac{-3(-4)}{2}x^2 + \frac{-3(-4)(-5)}{6}x^3 + \ldots = \boxed{1 - 3x + 6x^2 - 10x^3 + \ldots}$$

(iv) $(1-3x)^{1/3}$

Sol. $(1-3x)^{1/3}$

Put $b = -3x$ & $n = 1/3$ in Binomial series Formula,
$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots$$
we have :

$$(1-3x)^{1/3} = 1 + \left(\tfrac{1}{3}\right)(-3x) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-3x)^2 + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\left(\frac{1}{3}-2\right)}{3!}(-3x)^3 + \ldots$$

$$= 1 - x + \frac{1}{2}\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)(9x^2) + \frac{1}{6}\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)(-27x^3) + \ldots$$

$$= 1 - x - \frac{18}{18}x^2 - \frac{270}{162}x^3 + \ldots = \boxed{1 - x - x^2 - \frac{5}{3}x^3 + \ldots}$$

(v) $(4+x)^{1/2}$

(IA-2019)

Sol. $(4+x)^{1/2} = \left[4\left(1+\frac{x}{4}\right)\right]^{1/2} = 2\left(1+\frac{x}{4}\right)^{1/2}$

SOLUTION OF EXERCISE # 2.2

Put $b = \frac{x}{4}$ & $n = \frac{1}{2}$ in Binomial series Formula,

$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots$

we have :

$$=2\left[1+\left(\frac{1}{2}\right)\left(\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(\frac{x}{4}\right)^2+\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{x}{4}\right)^3+\ldots\right]$$

$$=2\left[1+\frac{x}{8}+\frac{1}{2}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{x^2}{16}\right)+\frac{1}{6}\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x^3}{64}\right)+\ldots\right]$$

$$=2\left[1+\frac{x}{8}-\frac{x^2}{128}+\frac{3x^3}{3072}+\ldots\right]=\boxed{2+\frac{x}{4}-\frac{x^2}{64}+\frac{x^3}{512}+\ldots}$$

(vi) $(2+x)^{-3}$

Sol. $(2+x)^{-3}=\left[2\left(1+\frac{x}{2}\right)\right]^{-3}=2^{-3}\left(1+\frac{x}{2}\right)^{-3}=\frac{1}{8}\left(1+\frac{x}{2}\right)^{-3}$

Put $b = \frac{x}{2}$ & $n = -3$ in Binomial series Formula

$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots$

we have :

$$=\frac{1}{(2)^3}\left[1+(-3)\left(\frac{x}{2}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-3)(-3-1)(-3-2)}{3!}\left(\frac{x}{2}\right)^3+\ldots\right]$$

$$=\frac{1}{8}\left[1-\frac{3x}{2}+\frac{(-3)(-4)}{2}\left(\frac{x^2}{4}\right)+\frac{(-3)(-4)(-5)}{6}\left(\frac{x^3}{8}\right)+\ldots\right]$$

$$=\frac{1}{8}\left[1-\frac{3x}{2}+\frac{12x^2}{8}-\frac{60x^3}{48}+\ldots\right]=\boxed{\frac{1}{8}\left[1-\frac{3}{2}x+\frac{3}{2}x^2-\frac{5}{4}x^3+\ldots\right]}$$

Q.2: Using the binomial expansion, calculate to the nearest hundredth.

SOLUTION OF EXERCISE # 2.2

(i) $\sqrt[4]{65}$

Sol. $\sqrt[4]{65}$

$$=(81-16)^{1/4}$$

$$=\left[81\left(1-\frac{16}{81}\right)\right]^{1/4}$$

$$=(81)^{1/4}\left(1-\frac{16}{81}\right)^{1/4}$$

$$=3\left[1+\left(\frac{1}{4}\right)\left(-\frac{16}{81}\right)+\ldots\right]$$

$$=3\left[1-\frac{16}{324}+\ldots\right]$$

$$=3[1-0.04+\ldots]$$

$$=3(0.95)=\boxed{2.84}$$

(ii) $\sqrt{17}$ (IIA-2015)

Sol. $\sqrt{17}$

$$=\sqrt{9+8}$$

$$=\sqrt{9\left(1+\frac{8}{9}\right)}$$

$$=3\left(1+\frac{8}{9}\right)^{1/2}$$

$$=3\left[1+\left(\frac{1}{2}\right)\left(\frac{8}{9}\right)+\ldots\right]$$

$$=3\left[1+\frac{4}{9}+\ldots\right]$$

$$=3[1+0.44+\ldots]$$

$$=3(1.44)=\boxed{4.33}$$

(iii) $(1.01)^{-7}$

Sol. $(1.01)^{-7}$

$$=(1+0.01)^{-7}$$

$$=1+(-7)(0.01)+\ldots$$

$$=1-0.07+\ldots$$

$$=\boxed{0.93}$$

(iv) $\sqrt{28}$ (IA-2022)

Sol. $\sqrt{28}$

$$=\sqrt{25+3}$$

$$=\sqrt{25\left(1+\frac{3}{25}\right)}$$

$$=5\left(1+\frac{3}{25}\right)^{1/2}$$

$$=5\left[1+\left(\frac{1}{2}\right)\left(\frac{3}{25}\right)+\ldots\right]$$

$$=5\left[1+\frac{3}{50}+\ldots\right]$$

$$=5[1+0.06+\ldots]$$

$$=5(1.06)=\boxed{5.30}$$

(v) $\sqrt{40}$

Sol. $\sqrt{40}$

$$=\sqrt{36+4}$$

$$=\sqrt{36\left(1+\frac{4}{36}\right)}$$

$$=6\left(1+\frac{1}{9}\right)^{1/2}$$

$$=6\left[1+\left(\frac{1}{2}\right)\left(\frac{1}{9}\right)+\ldots\right]$$

$$=6\left[1+\frac{1}{18}+\ldots\right]$$

$$=6[1+0.0556+\ldots]$$

$$=6[1.0556]=\boxed{6.30}$$

(vi) $\sqrt{80}$ (IIA-2018)

Sol. $\sqrt{80}$

$$=\sqrt{81-1}$$

$$=\sqrt{81\left(1-\frac{1}{81}\right)}$$

$$=9\left(1-\frac{1}{81}\right)^{1/2}$$

$$=9\left[1-\left(\frac{1}{2}\right)\left(\frac{1}{81}\right)+\ldots\right]$$

$$=9\left[1-\frac{1}{162}+\ldots\right]$$

$$=9[1-0.0062+\ldots]$$

$$=9[0.9938]=\boxed{8.94}$$

SOLUTION OF EXERCISE # 2.2

Q.3: Find the coefficient of x^5 in the expansion of:

(i) $\dfrac{(1+x)^2}{(1-x)^2}$ (IIA-2018)

Sol. $\dfrac{(1+x)^2}{(1-x)^2} = (1+x)^2(1-x)^{-2}$

$$= (1+2x+x^2)\left[1+(-2)(-x)+\frac{(-2)(-3)}{2!}(-x)^2+\frac{(-2)(-3)(-4)}{3!}(-x)^3\right.$$

$$\left.+\frac{(-2)(-3)(-4)(-5)}{4!}(-x)^4+\frac{(-2)(-3)(-4)(-5)(-6)}{5!}(-x)^5+\ldots\right]$$

$$= (1+2x+x^2)(1+2x+3x^2+4x^3+5x^4+6x^5+\ldots)$$

Eliminating all power of x except x^5, we get

$= 6x^5 + 10x^5 + 4x^5 = 20x^5$

Hence, coefficient of x^5 is : $\boxed{20}$

(ii) $\dfrac{(1+x)^2}{(1-x)^3}$

Sol. $\dfrac{(1+x)^2}{(1-x)^3} = (1+x)^2(1-x)^{-3}$

$$= (1+2x+x^2)\left[1+(-3)(-x)+\frac{(-3)(-4)}{2!}(-x)^2+\frac{(-3)(-4)(-5)}{3!}(-x)^3\right.$$

$$\left.+\frac{(-3)(-4)(-5)(-6)}{4!}(-x)^4+\frac{(-3)(-4)(-5)(-6)(-7)}{5!}(-x)^5+\ldots\right]$$

$$= (1+2x+x^2)(1+3x+6x^2+10x^3+15x^4+21x^5+\ldots)$$

Eliminating all power of x except x^5, we get

$= 21\,x^5 + 30\,x^5 + 10\,x^5 = 61\,x^5$

Hence, coefficient of x^5 is : $\boxed{61}$

SOLUTION OF EXERCISE # 2.2

Q.4: If 'x' is nearly equal to unity, prove that

$$\frac{mx^n - nx^m}{x^n - x^m} = \frac{1}{1-x}$$

(IA-2018), (IIA-2020)

Sol. Let x = (1 + h) (where h is very small)

$$\text{L.H.S} = \frac{mx^n - nx^m}{x^n - x^m} = \frac{m(1+h)^n - n(1+h)^m}{(1+h)^n - (1+h)^m}$$

$$= \frac{m\left[1 + nh + \frac{n(n-1)}{2!}h^2 + \ldots\right] - n\left[1 + mh + \frac{m(m-1)}{2!}h^2 + \ldots\right]}{\left[1 + nh + \frac{n(n-1)}{2!}h^2 + \ldots\right] - \left[1 + mh + \frac{m(m-1)}{2!}h^2 + \ldots\right]}$$

Neglecting h^2 and higher power of 'h', we get

$$= \frac{m(1+nh) - n(1+mh)}{(1+nh) - (1+mh)}$$

$$= \frac{m + mnh - n - mnh}{1 + nh - 1 - mh}$$

$$= \frac{m-n}{nh - mh} = \frac{m-n}{h(n-m)}$$

$$= \frac{m-n}{-h(m-n)}$$

$$= \frac{1}{-h} = \frac{1}{-(x-1)} = \frac{1}{1-x} = \text{R.H.S.}$$

PROVED